

# Fractions

## Pictures and written fractions

Here are four different ways of expressing/explaining a fraction

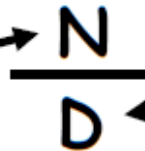
- $\frac{1}{2}$
- 1 out of 2
  - 1 divided by 2
- $\frac{1}{2}$
- 1 over 2
  - One half

It is important that students recognise these different representations to help them solve problems.



## Numerator and Denominator

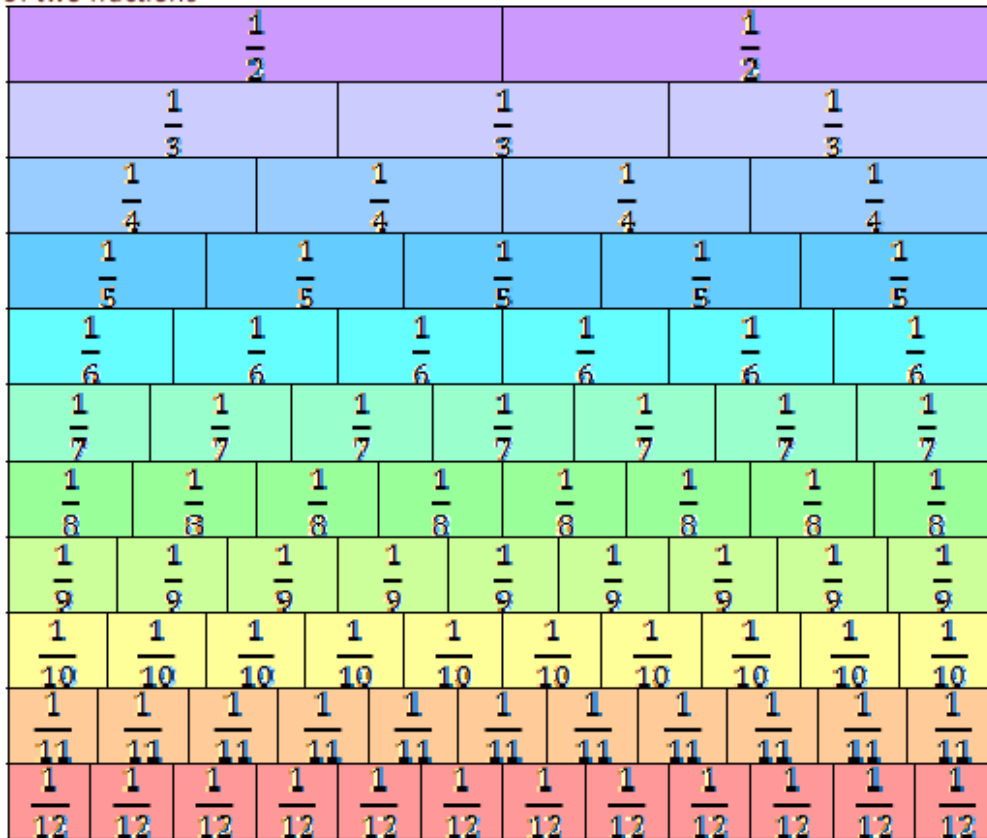
**Numerator**  
Number of pieces we are talking about



**Denominator**  
The Divisor: Number of equal pieces the amount is being divided into

## Fraction Wall to show equivalence

A fraction wall is often shown to help students understand equivalent fractions and to compare the size of two fractions



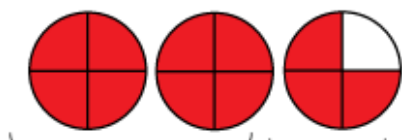
Be prepared to be flexible about fraction, decimal fraction and percentage equivalence.

## Mixed numbers/top heavy/improper/Proper

$\frac{3}{4}$  Is a proper fraction because the numerator is smaller than the denominator

$\frac{11}{4}$  Is an improper fraction because the numerator is bigger than the denominator

Sometimes we call this a top-heavy fraction



2 Wholes  
or  
8 quarters

3 quarters

$2\frac{3}{4}$  This is a mixed number with a whole number and a fraction

8 quarters

3 quarters

$\frac{11}{4}$  This is an improper/Top Heavy Fraction

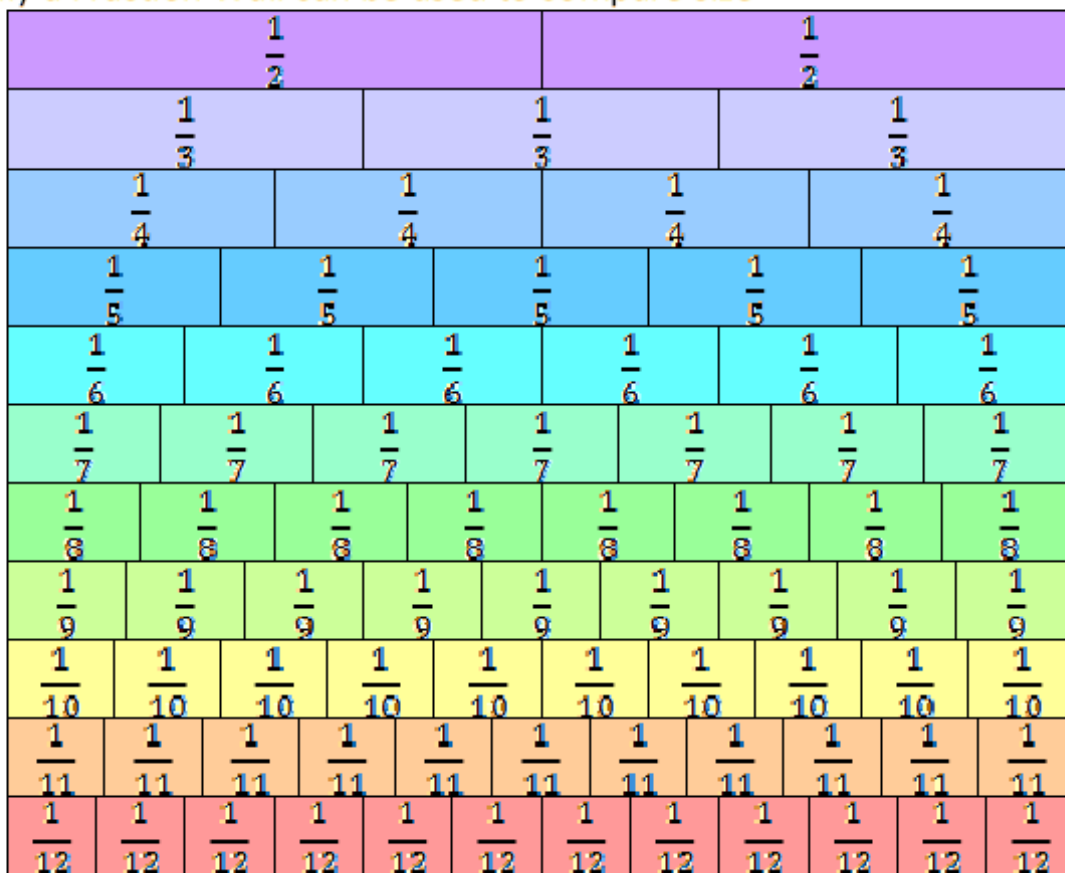
## Vocabulary

- Proper
- Improper
- Top Heavy
- Mixed
- Numerator
- Denominator
- Equivalent
- Common Denominator
- Simplify
- Integer
- Whole Number
- Factors
- Multiples
- Simplify Fully
- Cancelling
- Divide

Regular times table practice (with associated division facts)

## Ordering / Comparison

Initially a Fraction Wall can be used to compare size

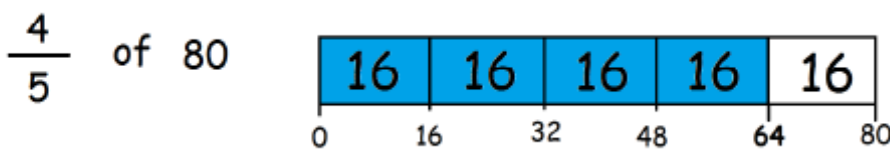


Then equivalent fractions can be found to compare directly

$$\begin{array}{cccc}
 \frac{1}{2} & \frac{3}{4} & \frac{3}{8} & \frac{7}{16} \\
 \downarrow & \downarrow & \downarrow & \downarrow \\
 \frac{8}{16} & \frac{12}{16} & \frac{6}{16} & \frac{7}{16}
 \end{array}$$

It is then much easier to compare the fractions as they all have the same denominator.

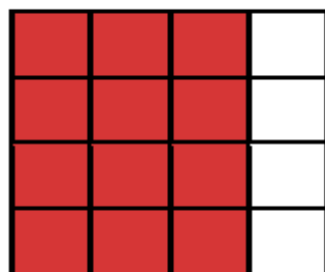
## Fractions of an amount



$$\frac{1}{5} \text{ of } 80 = 80 \div 5 = 16$$

$$\text{so } \frac{4}{5} \text{ of } 80 = 4 \times 16 = \boxed{\phantom{00}}$$

$\frac{3}{4}$  of 16



Shade in  
3 out of every 4  
boxes  
 $\frac{3}{4}$  of 16 is 12

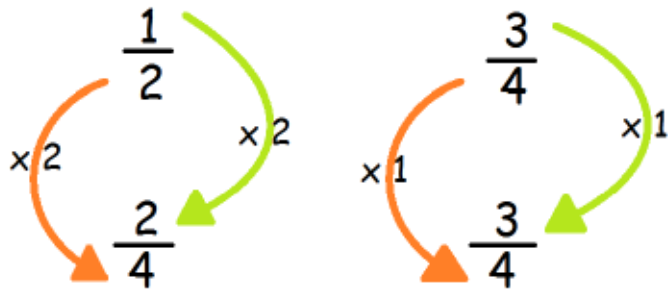
## Vocabulary

Proper  
Improper  
Top Heavy  
Mixed  
Numerator  
Denominator  
Equivalent  
Common Denominator  
Simplify  
Integer  
Whole Number  
Factors  
Multiples  
Simplify Fully  
Cancelling  
Divide

Regular times table practice (with associated division facts)

## Finding Equivalent Fractions

It is difficult to compare fractions unless they are alike and have a common denominator. This is a common multiple of all the denominators and is often the lowest common multiple (LCM).



When all fractions have a common denominator, numerators can be easily compared.

The same amount can be represented by different fractions.

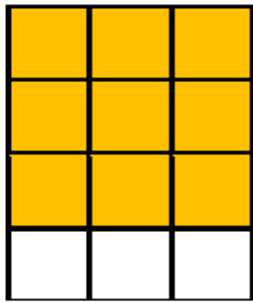
$$\frac{3}{4} = \frac{9}{12}$$

x 3 (above) and x 3 (below)

$$\frac{2}{3} = \frac{10}{15}$$

x 5 (above) and x 5 (below)

Using diagrammatic representation and splitting into different sized pieces.



9 out of 12 parts are shaded  
or you could say  
3 out of 4 rows are shaded



$$\frac{3}{4}$$



$$\frac{9}{12}$$

Pictures are used to reinforce proportionality. The same amount of the circle is shaded – but we can represent this with different fractions.

## Simplifying (inverse of equivalence)

Fractions can be simplified by looking for common factors and dividing by these numbers. It is possible to take different steps depending on which numbers we chose to divide with.

$$\frac{18}{30} = \frac{9}{15} = \frac{3}{5}$$

÷ 2 (above) and ÷ 2 (below) for the first step; ÷ 3 (above) and ÷ 3 (below) for the second step.

← No more common factors for 3 and 5 so we cannot simplify any further

This example has been simplified in one step → because the largest common factor of 32 and 80 was found to divide by (16 is a factor of both). The number of steps does not matter - it only speeds up the simplifying process.

$$\frac{32}{80} = \frac{2}{5}$$

÷ 16 (above) and ÷ 16 (below)

## Vocabulary

- Proper
- Improper
- Top Heavy
- Mixed
- Numerator
- Denominator
- Equivalent
- Common Denominator
- Simplify
- Integer
- Whole Number
- Factors
- Multiples
- Simplify Fully
- Cancelling
- Divide

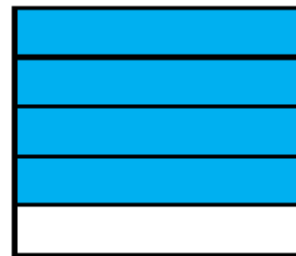
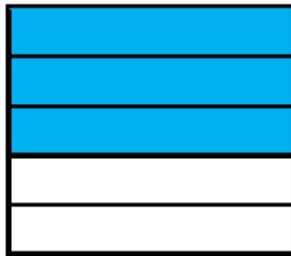
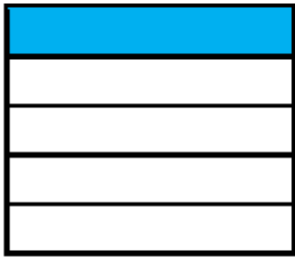
Regular times table practice (with associated division facts)

## Adding Fractions

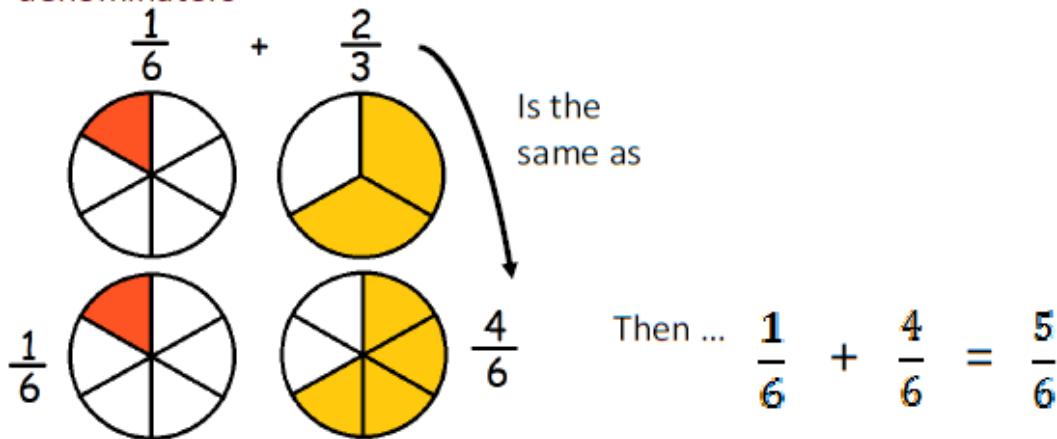
1) Diagrams with common denominators

$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$$

1 fifth + 3 fifths = 4 fifths



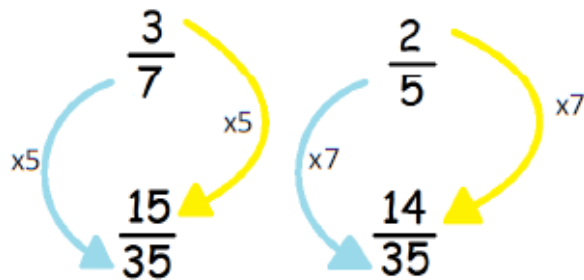
2) Diagrams to show conversion to equivalent fractions with common denominators



3) Finding common denominators

$$\frac{3}{7} + \frac{2}{5}$$

Equivalent fractions with a common denominator can be found in order to be able to add the fractions together.



This calculation becomes ...

$$\frac{14}{35} + \frac{15}{35} = \frac{29}{35}$$

If the answer is an improper fraction then it should be simplified as a mixed number.

### Vocabulary

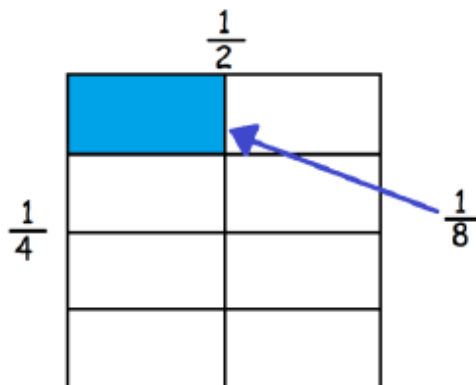
- Proper
- Improper
- Top Heavy
- Mixed
- Numerator
- Denominator
- Equivalent
- Common Denominator
- Simplify
- Integer
- Whole Number
- Factors
- Multiples
- Simplify Fully
- Cancelling
- Divide

## Multiplying Fractions

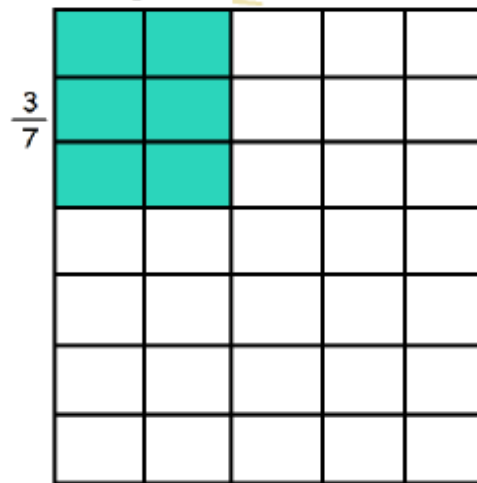
With diagrams

$$\frac{1}{4} \times \frac{1}{2}$$

"a quarter of a half"



$$\frac{3}{7} \times \frac{2}{5} = \frac{6}{35}$$



Without diagrams

$$\frac{2}{15} \times \frac{3}{8} = \frac{2 \times 3}{15 \times 8} = \frac{6}{120} = \frac{3}{60} = \frac{1}{20}$$

$$\frac{2}{15} \times \frac{3}{8} = \frac{2 \times 3}{15 \times 8}$$

Factors of the denominators:  
3 × 5 = 15    2 × 4 = 8

$$= \frac{\cancel{2} \times \cancel{3}}{\cancel{3} \times 5 \times \cancel{2} \times 4} = \frac{1}{5 \times 4} = \frac{1}{20}$$

Cancelling factors where possible before calculating.

## Vocabulary

Proper

Improper

Top Heavy

Mixed

Numerator

Denominator

Equivalent

Common Denominator

Simplify

Integer

Whole Number

Factors

Multiples


Simplify Fully

Cancelling

Divide

Regular times table practice (with associated division facts)

With mixed numbers

Remember ...  $2 \frac{2}{5} =$    $= \frac{12}{5}$

$$2 \frac{2}{5} \times 3 \frac{4}{7}$$

Convert first to avoid  $2 \times 3 + \frac{2}{5} \times \frac{4}{7}$  only


$$= \frac{12}{5} \times \frac{25}{7}$$

$$= \frac{\cancel{3} \times 4 \times \cancel{5} \times 5}{\cancel{5} \times 7} = \frac{12 \times 5}{7} = \frac{60}{7} = 8 \frac{4}{7}$$

cancel

Dividing Fractions

$3 \div \frac{1}{2}$



How many  $\frac{1}{2}$ 's in 3?

- How many  $\frac{1}{2}$ 's are in 1? **There are 2**
- How many  $\frac{1}{2}$ 's are in 3? **There are 6**

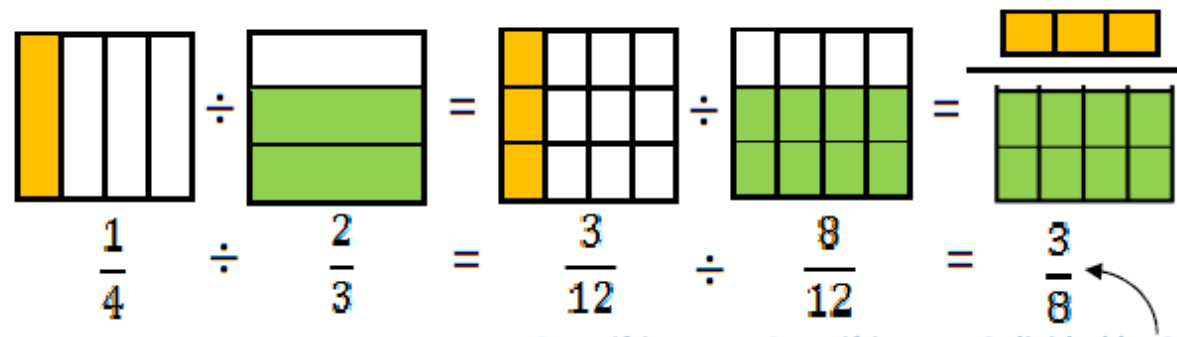
Use diagrams to help

$\frac{1}{2} \div \frac{1}{8}$       How many  $\frac{1}{8}$ 's in  $\frac{1}{2}$  ?

How many  $\frac{1}{8}$ 's are in 1 whole? **There are 8**

So ..... how many  $\frac{1}{8}$ 's are there in  $\frac{1}{2}$ ? **There are 4**

Dividing Using Common Denominators



$\frac{1}{4} \div \frac{2}{3} = \frac{3}{12} \div \frac{8}{12} = \frac{3}{8}$

3 twelfths ÷ 8 twelfths = 3 divided by 8

Dividing Using The Multiplicative Inverse

Multiplying by  $\frac{1}{2}$  is the same as dividing by 2

Dividing by  $\frac{1}{3}$  is the same as multiplying by 3

$\frac{2}{3} \div 4 = \frac{2}{3} \times \frac{1}{4}$  (Dividing by 4 is the same as multiplying by a quarter)

These are reciprocal/inverse/opposite relationships .....

opposite/inverse

$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{3 \times 5}{4 \times 2} = \frac{15}{8} = 1\frac{7}{8}$

reciprocal/inverse

Vocabulary

- Proper
- Improper
- Top Heavy
- Mixed
- Numerator
- Denominator
- Equivalent
- Common Denominator
- Simplify
- Integer
- Whole Number
- Factors
- Multiples
- Simplify Fully
- Cancelling
- Divide

Regular times table practice (with associated division facts)